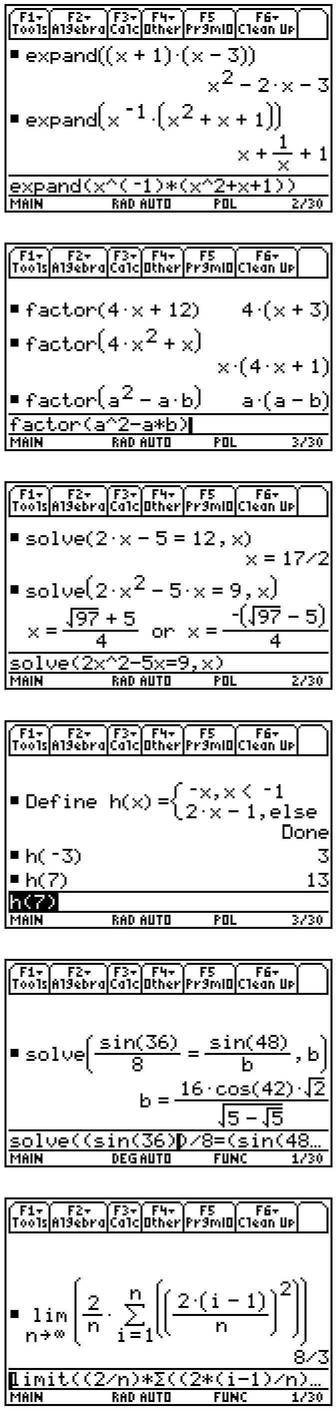


Northfield
Mount Hermon
School

Using the TI-89 in Mathematics

This manual was created by members of the Northfield Mount Hermon mathematics department in June of 2001. It is designed to supplement the instruction of mathematics in our curriculum and to serve as a guide to the student in using the TI-89 to enrich the learning of high school mathematics.

May 10, 2002

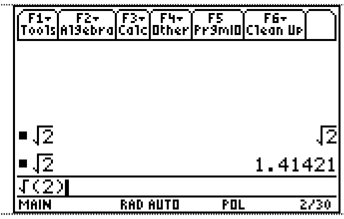
TABLE OF CONTENTS

CHAPTER 1 - OPERATIONS

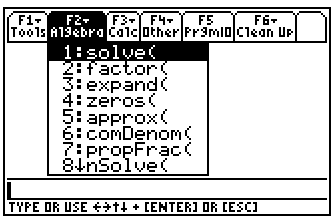
1. Preliminaries

arithmetic and other operations:
The TI-89 uses the standard keys for addition, multiplication, subtraction, division, exponentiation and grouping as do all calculators.

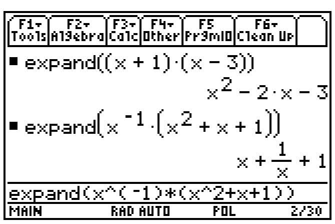
If the TI-89 returns a result in exact form, you can get a decimal approximation by pressing green diamond, Enter.

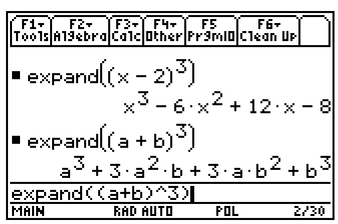


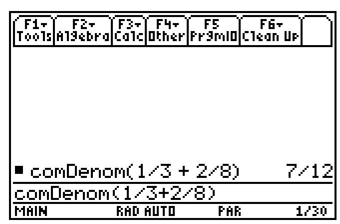
pull-down menus
Just like a computer, the TI-89 makes use of pull-down menus.
The expand command can be found using the F2, Algebra menu, shown below.



To type a variable name like a or b, precede the letter with the alpha key.
The following show some of the operations available under the F2:Algebra menu.



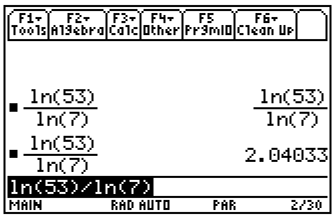




The only key that the TI-89 has for logarithms is for the natural logarithm. It is possible to access the common logarithm by typing **log** or by accessing it through the catalog.

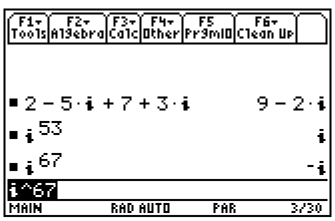
To compute the log for a number with a different base, it is necessary to use the change-of-base formula.

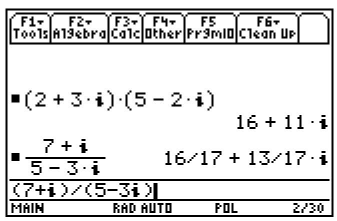
For example, if you wish to compute log7(53), you would need to enter ln(53)/ln(7). The following shows the exact result, and the result when green diamond ENTER is pressed.

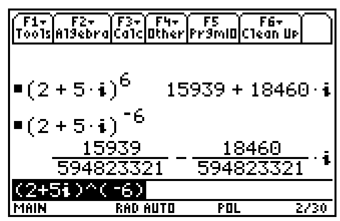


2. Complex numbers ***i*** is found above the catalog key

Operations on complex numbers are performed by using any of the arithmetic operators.

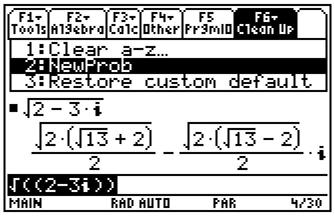


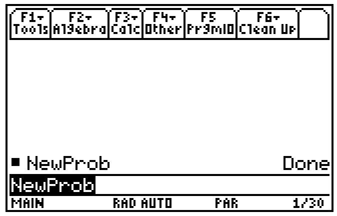




CHAPTER 2 - FACTORING

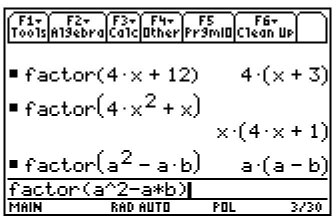
It is always a good idea to delete all one-character variables before beginning a new calculator session. The easiest way to do so is to press F6:Clean Up, 2:NewProb.



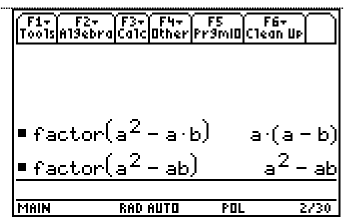


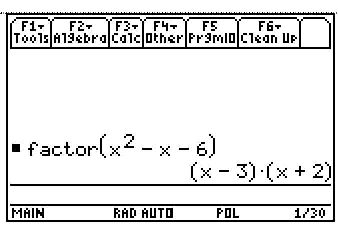
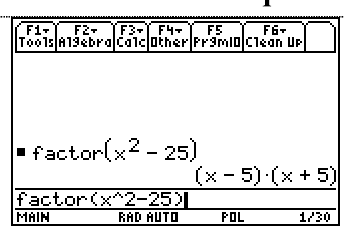
Here are examples of factoring:

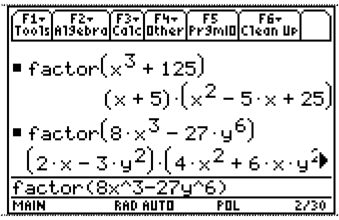
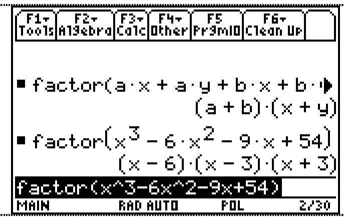
common term



NOTE WELL: It is necessary to write the multiplication symbol between the *a* and *b* variables. If it is left out, as it is in the second example to the right, the
TI-89 treats *ab* as a single variable and won't factor the expression.

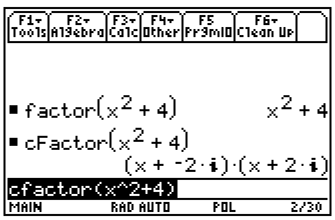


trinomials	**difference of two squares**
	

sum and difference of two cubes Note that to see the entire result below, it is necessary to arrow up and arrow to the right**.**	**grouping**
 	

Factoring over the complex numbers

In order to factor over the complex numbers, you must use the command **cfactor**.

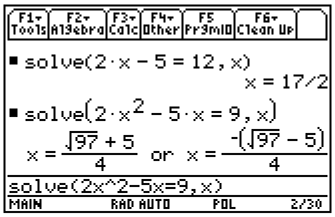


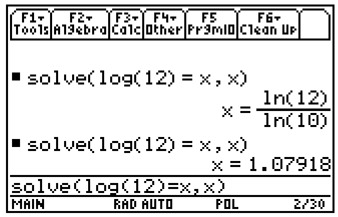
CHAPTER 3 - SOLVING EQUATIONS

1. linear, polynomial, logarithmic, exponential, rational, absolute value, radical.

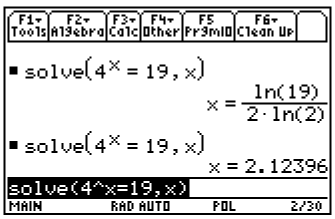
On the TI-89, if we wish to solve 2x-5=12 for x, we write
solve(2x-5=12,x)

Here are some examples of solving equations.

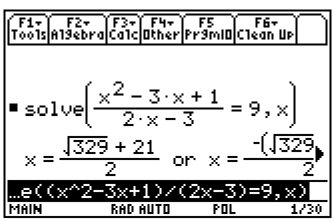


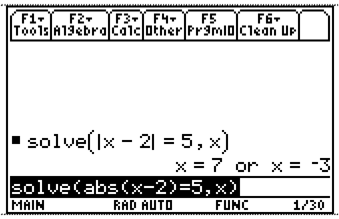
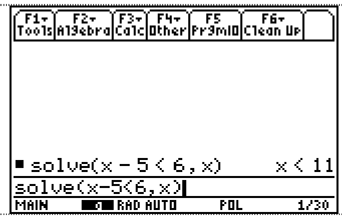


RECALL: Pressing green diamond and then Enter gives a decimal approximation.



The next example is a particularly difficult problem to do by hand that the TI-89 does in a snap.



<table>
<tr>
<td>Here is the solution to an absolute value problem. The absolute value command can be found under MATH (2nd 5), 1:Number, 2:abs, or it can be accessed by the catalog key.</td>
<td>Here we solve a linear inequality.</td>
<td>Here we solve a radical equation:</td>
</tr>
<tr>
<td>
</td>
<td>
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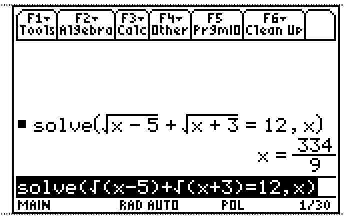
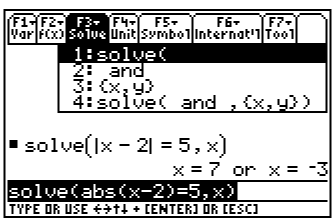
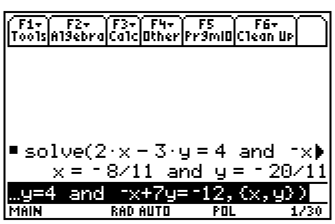
<td>
</td>
</tr>
</table>

2. Systems of Equations

This is the syntax necessary to solve a system of linear equations.
This command can most easily be accessed by CUSTOM (2nd Home), F3: Solve, 4: solve (and , {x,y}).



solve(2x -3y = 4 and –x + 7y = -12,{x,y})



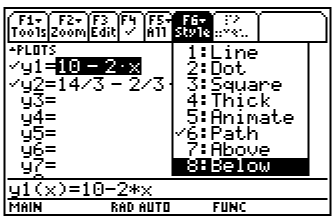
3. Linear Programming

Here is an example of a linear programming problem, and how to go about solving it.

Find the maximum value of the expression z = 5x + 8y, subject to the following restrictions:

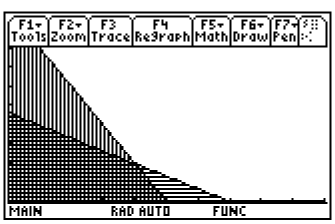
$$2x + y \le 10$$
$$2x + 3y \le 14$$
$$x \ge 0$$
$$y \ge 0$$

First, we graph the solution set. This requires solving each equation for y and shading the appropriate region. The two equations are y1=10-2x, and y2=14/3-2/3x.
For example, we wish to graph **below** the graph of y = 10 – 2x. We select that function, choose F6:Style, and option 8:Below.



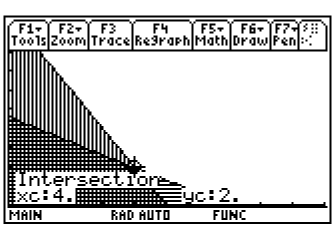
We do the same thing with all other functions to be graphed.
For xmin and ymin, we choose 0.

The resulting graph is shown below.

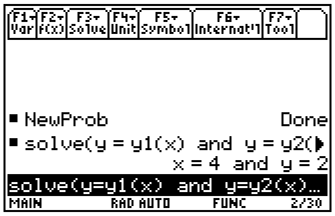


The technique of linear programming requires that we find all points of intersection and determine the value of z at each one.

We can find the coordinates of the points of intersection by F5:Math, 5:Intersection and following the prompts. The screen below shows that the key intersection point is at (4,2).



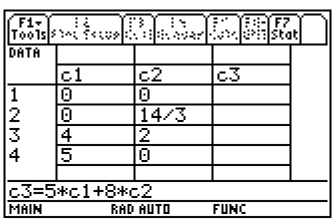
Alternately, we can use the command for solving a system of equations to find the intersection points, as shown below.
This command is **solve (y=y1(x) and y=y2(x), {x,y})**. Recall that the command **solve (and , {x,y})** is found by using the custom menu (2nd HOME) and pressing F3: Solve, and then choice 4: solve (and , {x,y}).

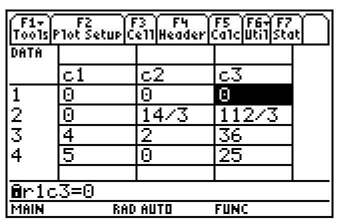


The other intersection points (which are x- or y-intercepts) can be easily found to be (0,0), (0,14/3) and (5,0).

So now the problem comes down to finding the maximum value of z = 5x + 8y over these four intersection points.

This can be done easily enough by hand, but we can also use the TI-89 as a spreadsheet to answer the question. This technique is covered in Chapter 8.



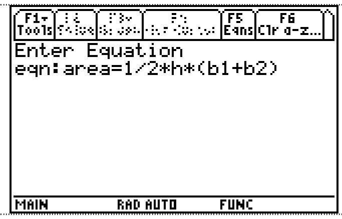


This table shows us that the maximum value of z occurs when x=0 and y=14/3.

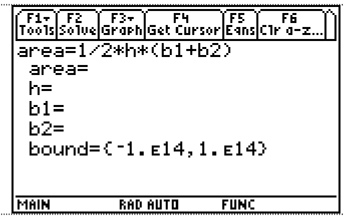
4. Using the Numeric Solver

The Numeric Solver is accessed through APPS, 9: Numeric Solver.

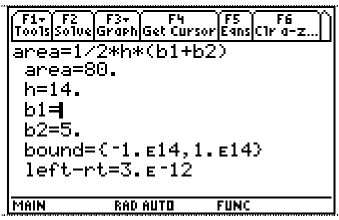
You will be prompted to enter an equation.



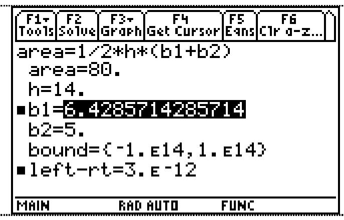
When you press ENTER, you will see the following screen.



Now you can enter data for all but one of the variable names, the one for which you wish to solve, which in this case is b1.

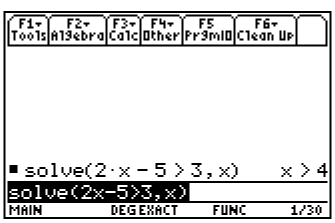


Place the cursor in the entry line for the unknown variable and press F2, Solve.

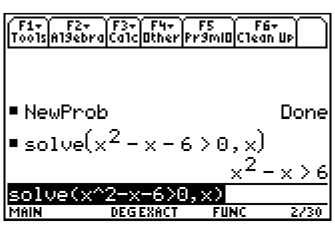


5. Solving non-linear inequalities

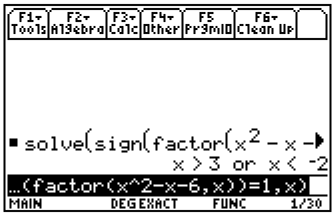
As the manual describes, the TI-89 cannot solve *non-linear* inequalities. It **can** solve *linear* inequalities, as the following shows:



However, any attempt to solve a non-linear inequality fails, as we see below:



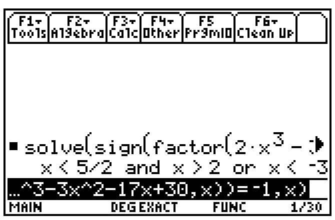
We can, however, "trick" the machine into giving the correct answer by use of the following command: solve (sign(factor(x^2-x-6,x))=1,x).
Note that this command gives the solution x>3 or x<-2.



The sign function delivers +1 when an expression is positive, and–1 if an expression is negative.

If we wish to solve 2x^3 – 3x^2 - 17x + 30 < 0, we would execute the command

solve (sign(factor(2x^3-3x^2-17x+30,x))= -1,x).

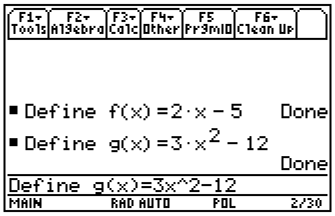


CHAPTER 4 - FUNCTIONS

1. Defining a function.

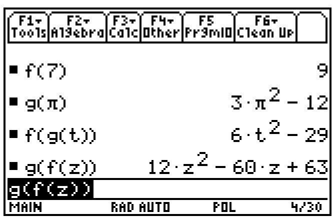
The Define command is found under the F4 menu.

Here are some examples.



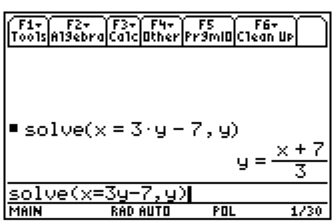
2. Evaluation and Composition of functions

Once functions have been defined, it is possible to evaluate them at particular values, and to compose them.



3. Finding the inverse of a function

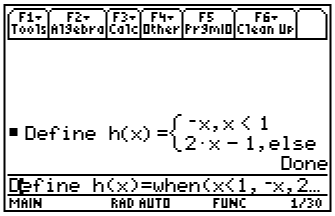
The inverse of the function y = 3x-7 is x = 3y-7. We can solve for y by executing the following command:

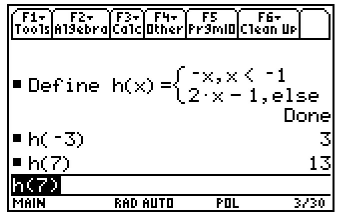


4. Piecewise definition of a function

If we wish to define the function $h(x) = \begin{cases} -x, x < 1 \\ 2x-1, x \geq 1 \end{cases}$, we use the following command:

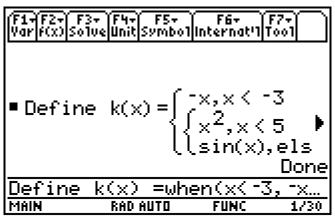
Define h(x) = when(x<1,-x,2x-1)
The word "when" must be typed from the keyboard, or accessed from the catalog.



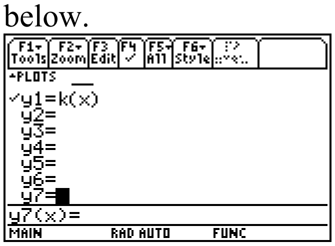


If we wish to define $k(x) = \begin{cases} -x & x < -3 \\ x^2 & -3 \le x < 5 \\ \sin(x) & x \ge 5 \end{cases}$, we execute the following command:

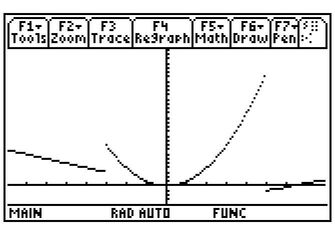
Define k(x) = when (x<-3, -x, when (x<5, x^2, sin(x))).

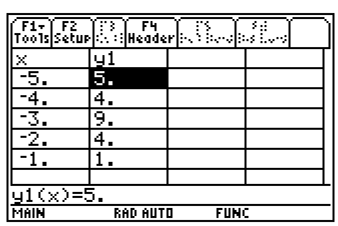


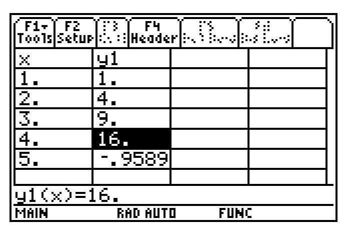
Once k(x) has been defined as above, it can be assigned to y1 by entering k(x) as shown below.



Once this has been done, it is a simple matter to graph k(x) and view a table of values.







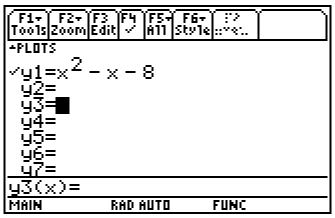
CHAPTER 5 - GRAPHING

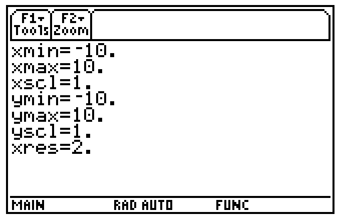
Obviously, the TI-89 **graphing** calculator is well-suited for graphing all kinds of functions, and even relations which aren't functions.

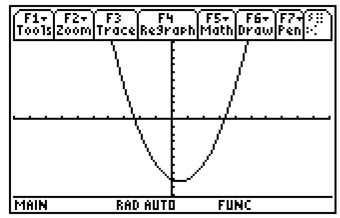
1. Graphing functions

A function is entered in the Y= window, and the appropriate window is entered.

Here are some examples:



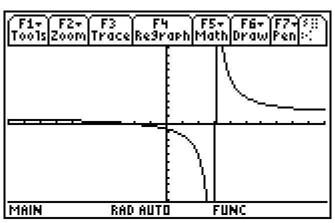




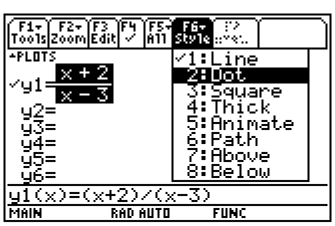
Here the window is the standard window, which can be set by entering Zoom, 6:Standard.

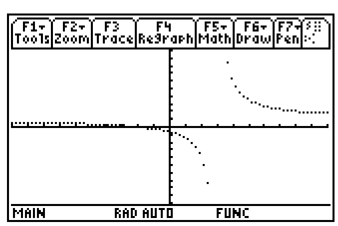
When a rational function is graphed, a line that **seems** to be the asymptote is drawn, because consecutive points are connected.

The following is the graph of y=(x+2)/(x-3):



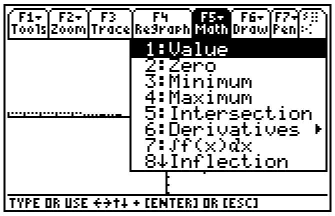
To eliminate the "line", which is often misunderstood to be a vertical asymptote, go to the Y= Editor, highlight the function y1, select menu F6, Style, and choose 2:Dot. The resulting graph is shown below to the right.





2. Maximum and minimum values; zeros

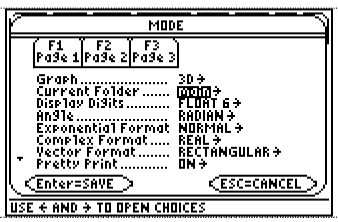
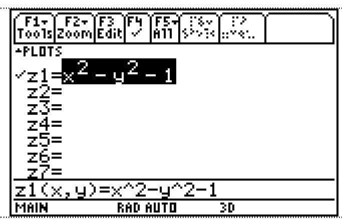
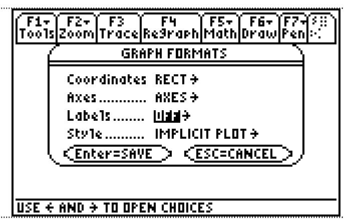
Just like other models of the TI, the TI-89 can find the value at any point, and the maximum and minimum values of a function that has been graphed. The commands for this are found under F5, Math, and are pictured below.

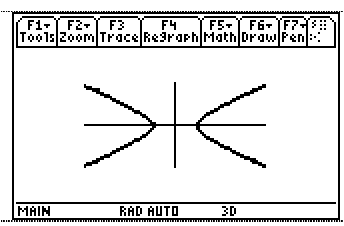


3. Graphing Relations

To graph a relation which cannot be expressed as y=f(x), we need to use the *implicit plot* feature of the TI-89.

Suppose we wish to graph $x^2 - y^2 = 1$

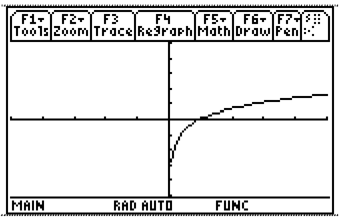
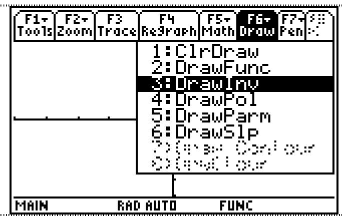
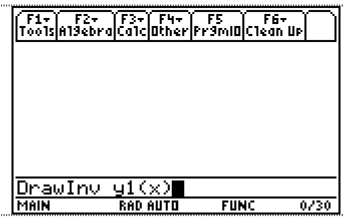
Press Mode and set Graph to 3D.	Press Y= and define the function $z1(x,y)=x^2-y^2-1$	Press F1:Tools, 9:Format and change the Style to Implicit Plot.
		

Press Graph to graph the relation. It will take some time for the data to be processed before the graph appears.	

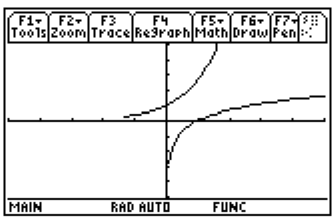
4. Graphing the inverse of a function

We saw before how we can **algebraically** find the inverse of a function.
Here we will see how to **graph** the inverse of a function.

Let's suppose we wish to draw the inverse of y=ln(x).

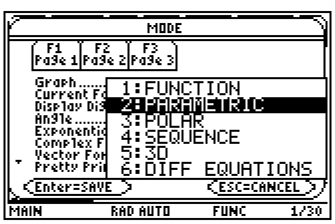
Enter ln(x) for y1 and graph it.	Now we go to F6:Draw and select 3:DrawInv	Pressing Enter returns us to the Home Screen, and prompts us for the function. Enter y1(x).
		

Pressing Enter gives us the desired result.

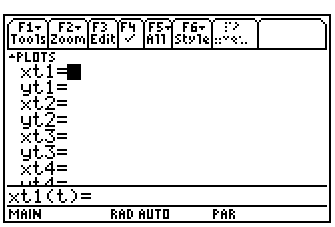


5. Graphing Parametric Relations

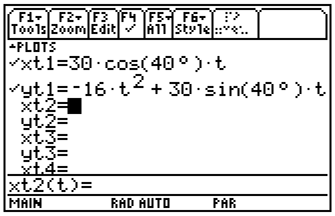
To graph parametric equations, the mode must be set to parametric.
Press the MODE key, choose Graph, and then 2:PARAMETRIC.



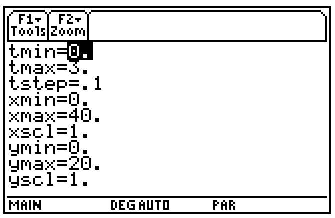
When you go to Y= at this point, you will be presented with pairs of equations that correspond to parametric equations.

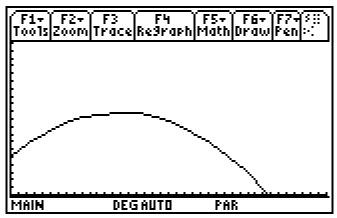


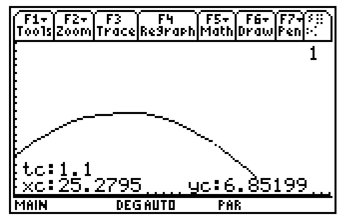
Enter a pair of parametric equations. Note that the independent variable is t.
An example is shown below. These are the equations that describe the path of a ball that is thrown with an initial velocity of 30 fps at an angle of 40 degrees to the horizontal, from an initial height of 5 ft.



Shown below is an appropriate window, and the resulting graph. Note the effect of choosing F3:Trace. Not only are you given the x- and y-coordinates, but also the corresponding value of time t.



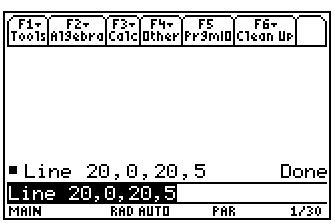


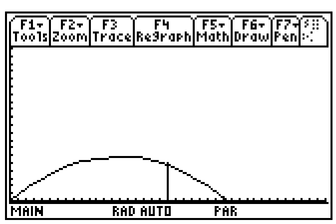


In many of these problems involving projectile motion, we wish to know if a projectile clears a wall. We can draw a wall (actually, a line) by the following command issued from the home screen:

Line 20,0,20,5

This draws a line from the point (20,0) to the point (20,5), which represents a 5 foot wall 20 feet from the launch of the projectile.





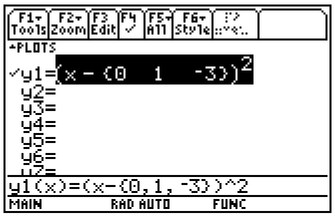
Close examination shows that the projectile will not clear the wall.

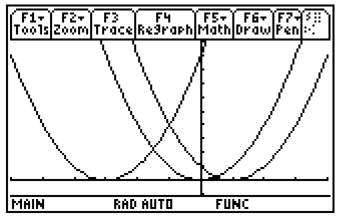
6. Graphing multiple functions

It is possible to write a function in the Y= Editor so that several related graphs are produced. For example, suppose we wished to graph $y = x^2$, $y = (x-1)^2$ and $y = (x+3)^2$.

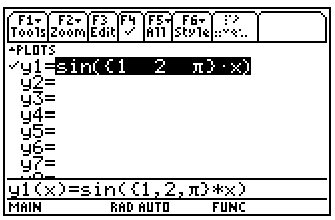
We could graph all three at once by assigning to y1 the expression (x-{0,1,-3})^2.

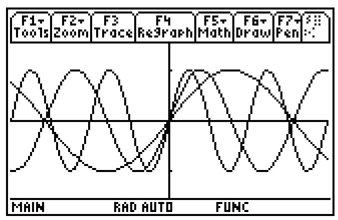
This is shown below.





Here is another example, using the sine curve.

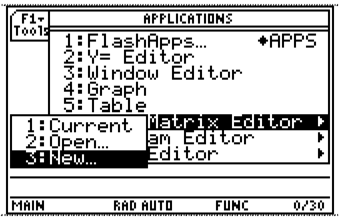
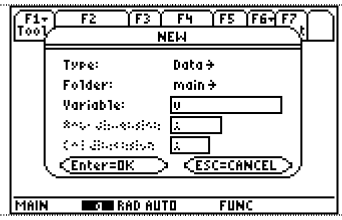
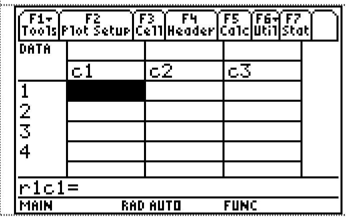




The resulting graphs may be difficult to see as completed graphs, but as they are being graphed, it is easy to see the effect of the multiplier on x.

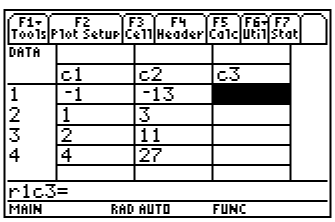
CHAPTER 6 - REGRESSION

When doing regression problems, you will be using the Data/Matrix Editor, and entering the ordered pairs into the editor in the *Data* type.

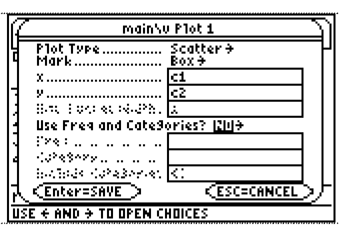
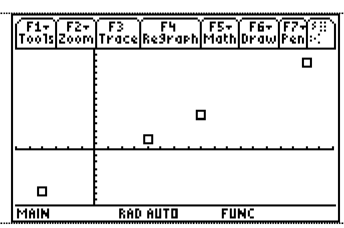
Press Apps, 6:Data/Matrix Editor, 3:New.	Leave the Type: as Data, and enter a simple one-variable name for the variable, such as v.	This brings up the following window, into which we can enter the data points.
		

1. Linear regression

We'll enter data that will be linear.

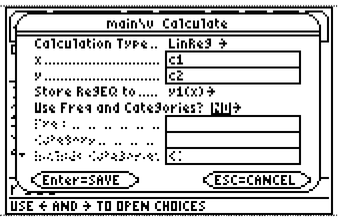
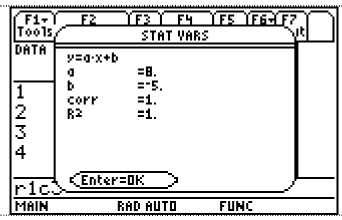
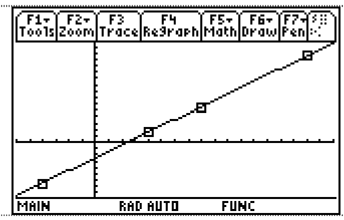


Before we perform linear regression, we can plot the data.

Press F2: Plot Setup and then F1: Define. For Plot Type: select 1:Scatter. For "x", enter c1, and for "y", enter c2 by typing the letter c followed by either 1 or 2.	To plot the data points, press green diamond F2, and select F2:Zoom. Select 9:ZoomData and the following will be graphed. You can trace the points and see the specific values.
	

This certainly looks like a linear relationship.

<table>
<tr>
<td>To perform the linear regression, press Apps, 6:Data/Matrix Editor, and 1:Current to return to the data points.

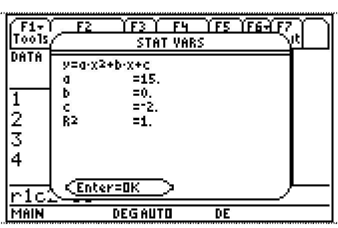
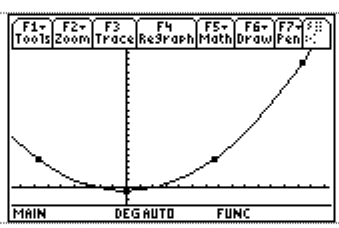
Press F5:Calc and pick 5:LinReg for the Calculation Type:
Enter c1 for x and c2 for y, and select y1(x) for Store RegEq to:</td>
<td>This will produce the following data, showing that the equation of the regression line is y = 8x – 5. The closer corr or R^2 is to 1 or –1, the better the "fit" is.</td>
<td>We can now graph and see how the line fits the data points.</td>
</tr>
<tr>
<td>
</td>
<td>
</td>
<td>
</td>
</tr>
</table>

2. Quadratic regression

Consider the following ordered pairs:

{(-1,13), (0,-2), (1,13), (2,58)}

<table>
<tr>
<td>We'll do everything we did before, except that we will select QuadReg for the Calculation Type.

Doing so gives the following equation, which we will store in y1:</td>
<td>Graphing the data points and the curve shows the goodness of fit.</td>
</tr>
<tr>
<td>
</td>
<td>
</td>
</tr>
</table>

3. Cubic regression

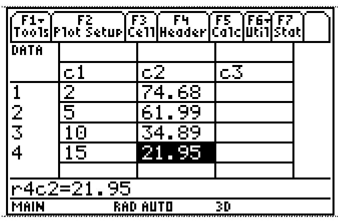
Cubic regression is done the same way as quadratic regression.

4. Exponential regression
The following is an example that demonstrates *Newton's Law of Cooling*, which states that the change in the temperature of an object is directly proportional to the difference between the temperature of the object and the surrounding medium.

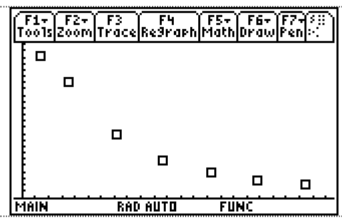
A thermometer was removed from a cup of hot water and placed in a tub of water with temperature of 5°C. The data in the following table was collected over the next thirty seconds, where Temp is the temperature of the thermometer.:

time	2	5	10	15	20	25	30
Temp	74.68	61.99	34.89	21.95	15.36	11.89	10.02

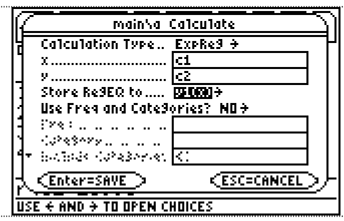
Here we enter the data into two lists.



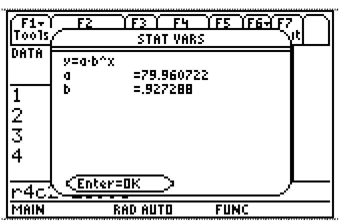
Plotting the data points give us the following:



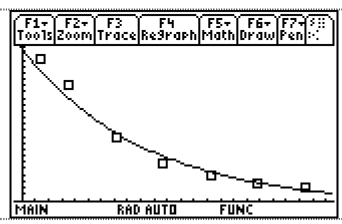
Doing the regression call to ExpReg gives the following



The regression equation is shown below:



Graphing the regression equation is shown next:



Once we have performed this regression, we can answer different types of questions, such as "What was the temperature of the cup of hot water?"

5. Other types of regression
The TI-89 can perform the following types of regression:
linear, quadratic, cubic, quartic, sine, exponential, logarithmic and logistic.

6. Summary of Regression Technique
- Apps, Data/Matrix Editor, Data
- Enter data points
- Plot the data points: F2:Plot Setup, F1:Define, F2:Zoom, 9:ZoomData
- Do the regression and save equation: Data/Matrix Ediotor, F5:Calc, specify kind of equation

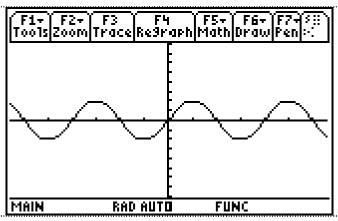
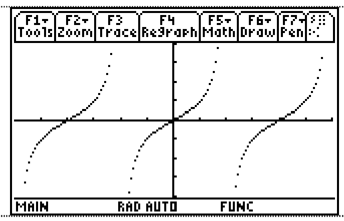
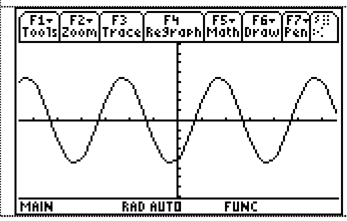
CHAPTER 7 - TRIGONOMETRY

The TI-89 has the cosine, sine and tangent functions built in. These are enough to generate the remaining three trigonometric function..

1. Graphing Trigonometric Functions

Graphing trig functions works the same as graphing any function.
It is best to select radian measure under Mode.
Zoom 7 gives trig values for the window.

Examples:

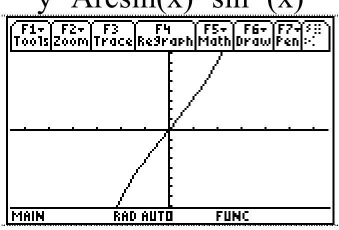
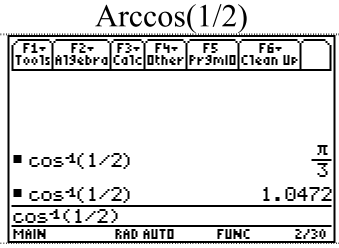
y=sin(x)	y=tan(x)	y=sin(x) - 2*cos(x)
		

The style of y=tan(x) was set to dot to avoid the jagged connections between points on either side of a vertical asymptote.

2. Inverse Trig Functions

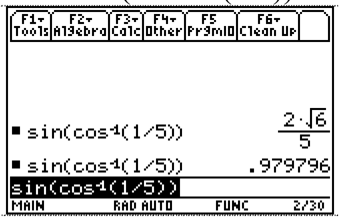
The inverse trig functions are located on the same keys as the trig functions.

Examples:

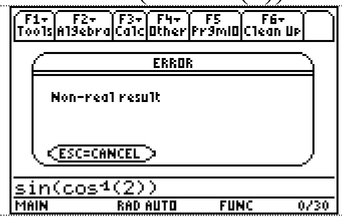
y=Arcsin(x)=sin^{-1}(x)	Arccos(1/2)
	

3. Evaluating compositions of trig and inverse trig functions

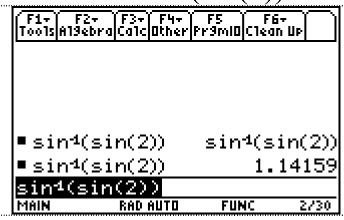
sin(Arccos(1/5))



sin(Arccos(2))



Arcsin(sin(2))

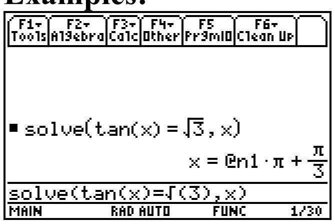


The last example is an important one, since it indicates that the TI-89 understands the appropriate range of the Arcsin function.

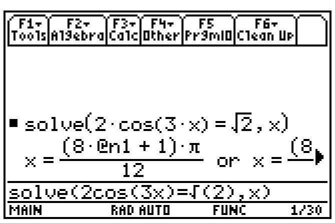
4. Trigonometric Equations

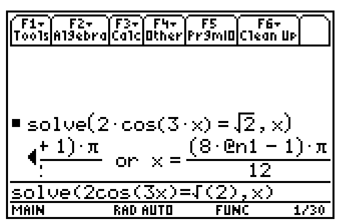
Solving equations with trig functions works the same as solving any kind of function.

Examples:

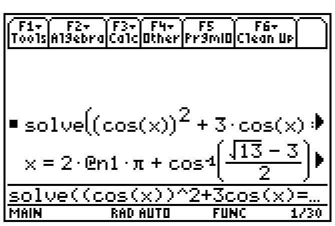


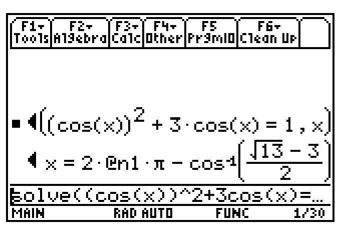
NOTE WELL*: The statement $x = @n1\pi + \pi/3$ means that solutions are of the form $\pi/3 + k\pi$, where k is an integer. @n1 represents any integer value*. For successive calculations, the number following n increases by 1, such as @n2, @n3, etc.





Here is the solution of $\cos^2(x) + 3\cos(x) = 1$

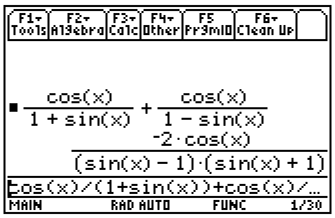


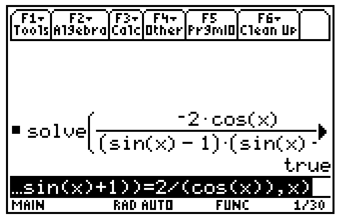


5. Trigonometric Identities
While work with identities must continue to be done by hand to ensure appropriate comprehension of key trig ideas, the TI-89 can help to validate an identity.

Here are some examples:

Show that $\dfrac{\cos(x)}{1+\sin(x)} + \dfrac{\cos(x)}{1-\sin(x)} = 2\sec(x)$

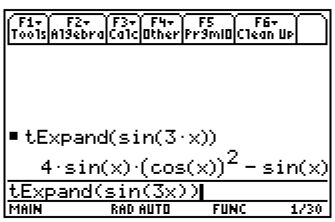


or


Also, we can graph each of these on the same axes, using different styles for each, to help convince us that the identity is indeed true.

Show that sin (3x) = 4 sin(x) $\cos^2$(x) – sin(x)

We use the command *texpand*, short for trig expand.

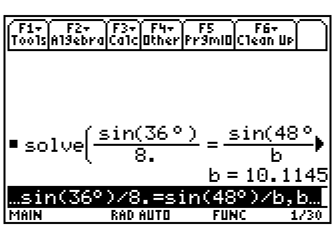


6. Law of Sines and Cosines

The TI-89 can be used in these type of problems very easily, since all that is really required is solving an equation.

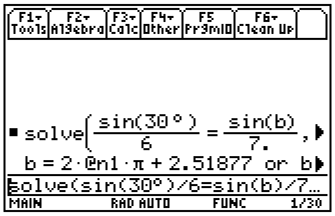
Here are some examples for the Law of Sines:
Find side b given that side a = 8, angle A = 36 degrees and angle B = 48 degrees.

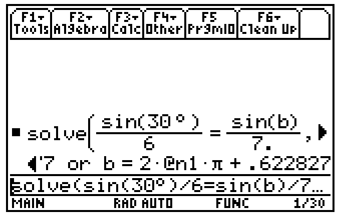
We want to solve $\dfrac{\sin(36)}{8} = \dfrac{\sin(48)}{b}$ for b. Note the use of the angle symbol, and that we type 8. to force a decimal approximation for the answer.



Here is an example of the ambiguous case, for which the TI-89 conveniently delivers both of the results.

Find angle b if side a = 6, side b = 7, and angle A = 30.

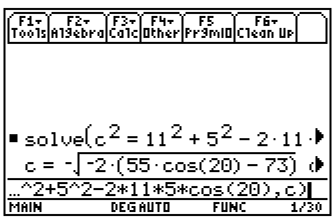


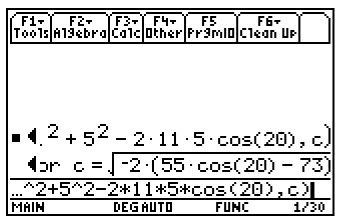


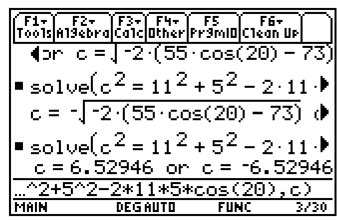
Although we get general solutions to this question, in the context of the problem we can only accept angle measures between 0 and π. So the two solutions are 2.5187 and .62282.

Here is an example that uses the Law of Cosines:

Find side c if side a = 11, side b = 5 and angle C = 20.
We execute the command solve (c^2=11^2+5^2-2(11)(5) cos(20),c).







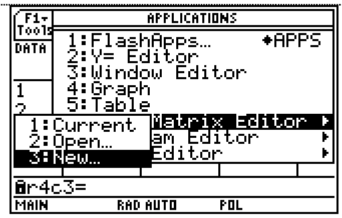
Here we need to eliminate the second, extraneous solution, since it is a negative number.

CHAPTER 8 - Table of Values ; Sequences and Series

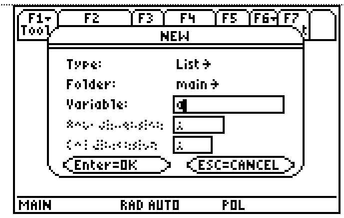
1. Table of Values

It is possible to use the TI-89 as a spreadsheet.

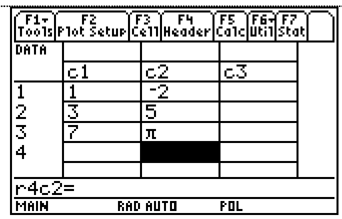
Press Apps, the 6:Data/Matrix Editor, then 3:New.



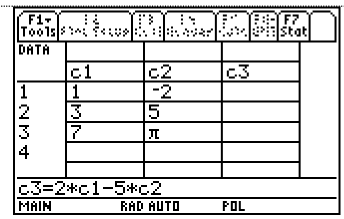
Then for Type: choose List, and name the variable with something simple, like *a*.



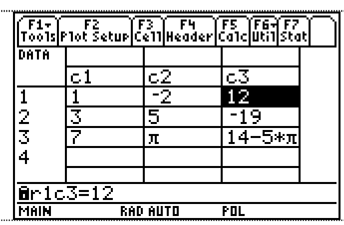
Enter values for columns c1 and c2.



To enter a formula for column c3, move the cursor to the cell labled c3, and type the formula for that column, which in this case is 2*c1-5*c2.



Pressing Enter fills the c3 column with the appropriate values.



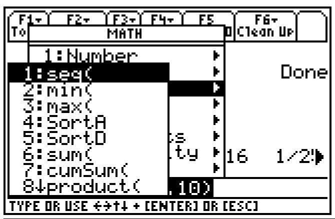
2. Sequences and Series

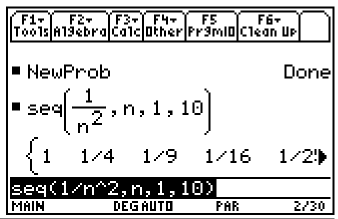
Suppose we wish to generate the following sequence on the TI-89:

$1/1^2, 1/2^2, 1/3^2, ..., 1/10^2$.

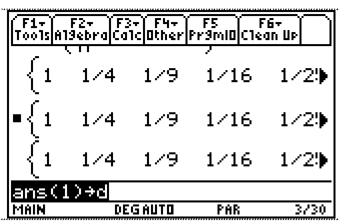
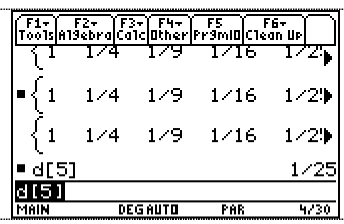
We use the seq command, found by going to MATH (found by pressing 2nd 5), 3:List, 1:seq

Here is how we would enter the sequence {1/n2}. for n = 1 to 10:

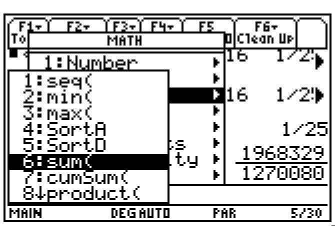


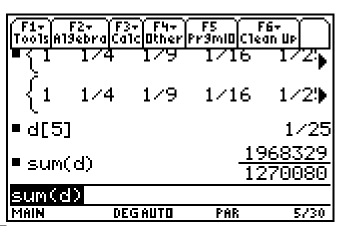


The result is enclosed in braces, which indicates that it is a *list*.

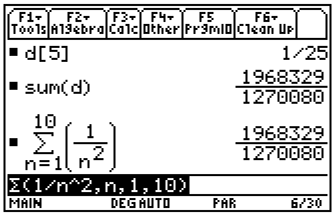
We can store the list into some variable name, say d, by pressing the STO> key.	Since this information is in a list named d, we can access individual elements by a call such as d[5]. Note the use of brackets.
	

To get the **sum** of the associated series $1/1^2 + 1/2^2 + 1/3^2 + ... + 1/10^2$, we can issue the command sum(d). the command sum is found by MATH, 3:List, 6:sum.

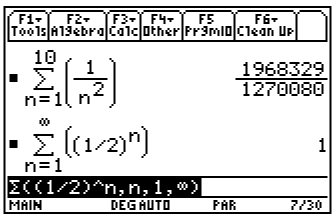




Alternately, we can use the $\sum$ command found under F3:Calc.



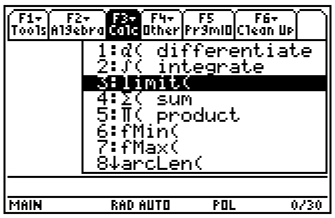
This command can also be used to find the sum of an infinite series. The infinity symbol is entered by pressing green diamond catalog.



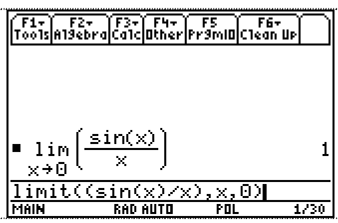
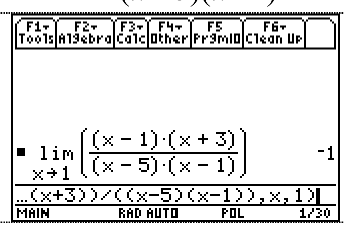
CHAPTER 9 - LIMITS

The TI-89 can be used to handle questions that involve limits.
To compute a limit, state the function and the value at which the limit is to be found.

The limit command is found under F3:Calc, option 3:limit(

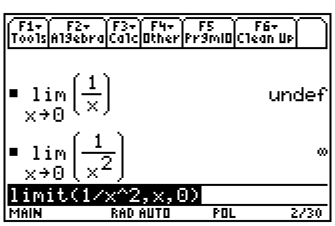


Examples:

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$	The following example illustrates how to find the y-value at a "hole" or removable discontinuity of a rational function. $$\lim_{x\to 1}\frac{(x-1)(x+3)}{(x-5)(x-1)}$$
	

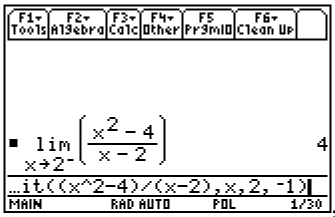
Note the difference between the following two non-existent limits.

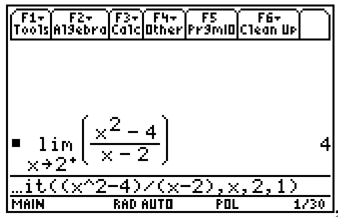
$$\lim_{x\to 0}\frac{1}{x} \text{ and } \lim_{x\to 0}\frac{1}{x^2}$$



We can also use the TI-89 to find left and right hand limits.
For a left-hand limit, simply use as the last parameter any negative number; for a right-hand limit, use any positive number.

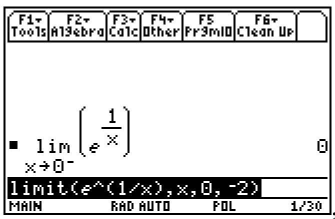
$$\lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} \qquad \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

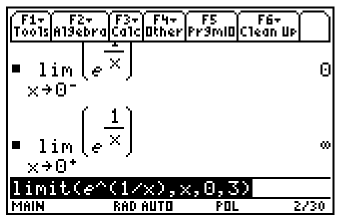


,


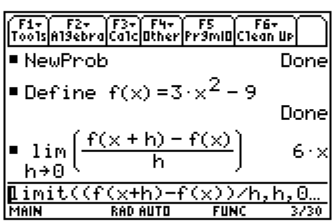
,

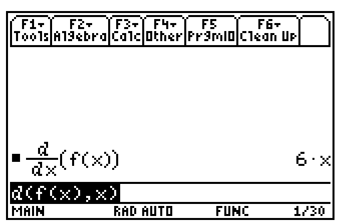
$$\lim_{x \to 0^-} e^{1/x} \qquad \lim_{x \to 0^+} e^{1/x}$$



,


For a final example, consider the following concept from calculus.
Define f(x) = $3x^2$-9, and evaluate the limit of (f(x+h)-f(x))/h as h approaches 0. This is the definition of the derivative, and this result should agree with the derivative of f(x).



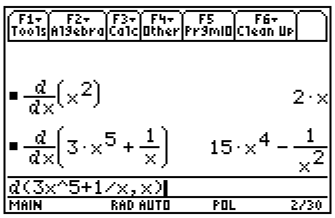


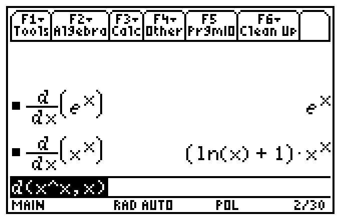
CHAPTER 10 - CALCULUS

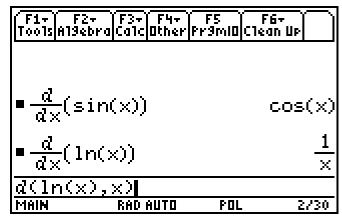
The TI-89 is well-suited to provide additional insight into Calculus, by graphical and numerical methods.

1. Differentiation

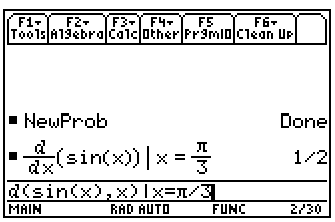
The following examples show the notation needed to find a derivative. The derivative key is found by accessing 2^{nd} 8, or F3:Calculus, 1:d(differentiate.







It is also possible to compute a derivative, and evaluate it at a specific point.
The following example shows how we can find the derivative of sin(x) and evaluate it at $x = \pi/3$, giving the result of 1/2.

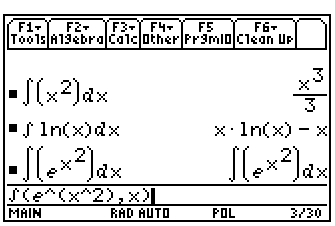


2. Integration

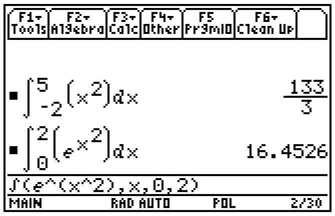
The integral key is found by accessing 2^{nd} 7, or F3:Calculus, 2:integrate.

Definite and indefinite integrals (or antiderivatives) are easily found. (Note that the constant term, C, is not printed in the answer for an indefinite integral.)

Note that in the last example the TI-89 simply returns the input since there is no antiderivative of e^{x^2}.

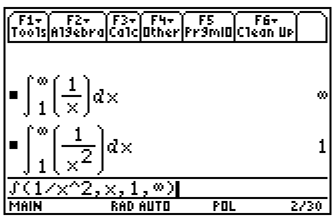


To compute a definite integral, add the left and right bounds as the last two parameters, as shown below.



The above example illustrates that although there is no antiderivative of e^{x^2}, it is possible to approximate the area under its curve between two fixed values.

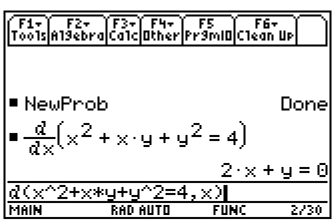
The TI-89 can also handle improper integrals:



3. Implicit differentiation

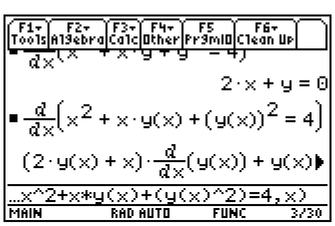
Suppose we wish to find dy/dx given the relation $x^2 + xy + y^2 = 4$.

If we execute the command **d(x^2+x*y+y^2=4,x)**, the result obtained is 2x + y, as shown below.

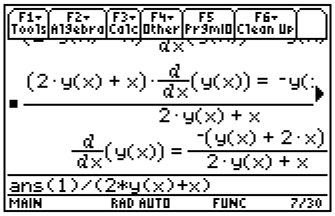


This incorrect result results because the TI-89 treats the variable **y** as a constant.
We can override this assignment by writing y(x) for y, which forces the TI-89 to think of this as a function of x, rather than a constant.

This returns the result shown below, which is (2y(x)+x) d/dx (y(x)) +y(x)+2x=0

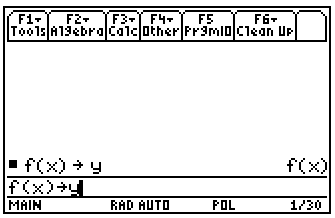


To solve for d/dx(y(x)), from each side we subtract 2x, subtract y(x), and divide the result by (2y(x)+x). This gives the result –(y(x)+2x)/(2y(x)+x) , as shown below.



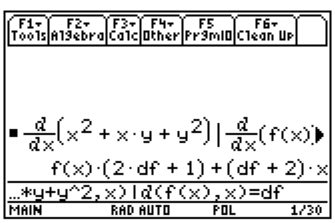
Alternately, we can perform the same operation by the following steps:

Store f(x) in y. This assigns the expression f(x) to y, which is necessary because the TI-89 has no way of knowing that y depends on x.

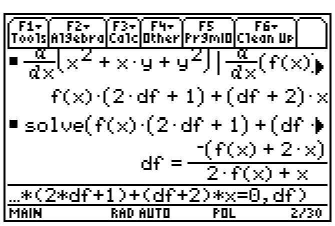


We now differentiate both sides of the equation with respect to x, by entering
d(x^2+x*y+y^2 = 4,x)|d(f(x),x)=df

The last part of the above command (|d(f(x),x)=df) defines dy/dx to be df, so that in the result, we will have the expression df instead of dy/dx.



If we solve for df we get the expression for dy/dx, which we are referring to as df:



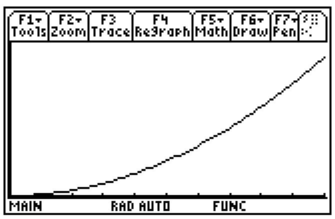
4. Riemann Sums

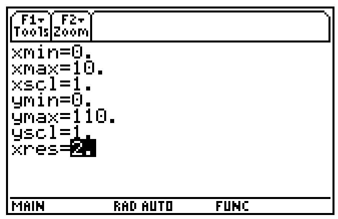
To draw an appropriate diagram illustrating Riemann Sums, we need to use some low-level programming commands, including *For*, *Line* and *EndFor*. These commands are most easily accessed through the catalog key.

The following command line, shown in bold, **For i,1,10 : block(i) : EndFor** executes whatever is in **block**, starting with i = 1 and ending with i = 10.

The command Line a,b,c,d draws a line segment with one endpoint at (a,b) and the second endpoint at (c.d).

Let's consider the curve $y=x^2$, with the window as shown. Enter $y = x^2$ into y1.



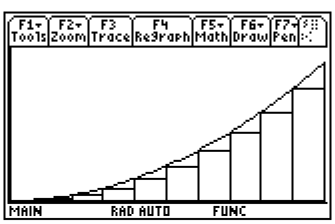


We can draw left-hand rectangles by issuing the following commands from the Home screen:

For i,1,10:Line i,0,i,y1(i):EndFor *this draws vertical line segments*

For i,0,9:Line i,y1(i),i+1,y1(i):EndFor *this draws horizontal line segments*

The resulting diagram looks as follows:



Now suppose we wish to evaluate $\int_0^2 x^2 dx$ with left-hand Riemann sums, with a partition of *n* subintervals. We'll represent this sum by Ln.

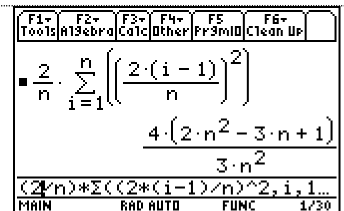
Each base has length 2/n. The height of the first rectangle is 0^2, the height of the second rectangle is $(2/n)^2$, the height of the third rectangle is $(4/n)^2$, the height of the i^{th} rectangle is $(2(i-1)/n)^2$, and the height of the last rectangle is $(2(n-1)/n)^2$.

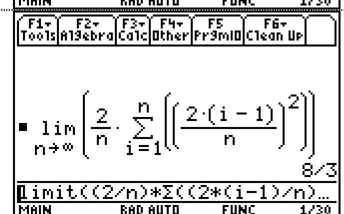
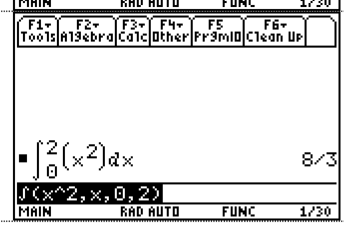
Thus the expression for Ln is $\frac{2}{n}\sum_{i=1}^{n}\left(\frac{2(i-1)}{n}\right)^2$.

We can get an **exact** expression for $\int_0^2 x^2 dx$ by letting n approach infinity in the above expression for Ln.

That is, $\int_0^2 x^2 dx = \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left(\frac{2(i-1)}{n}\right)^2$.

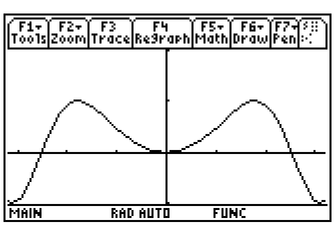
We can illustrate this work using the TI-89.

<table>
<tr><td>First we create an expression for Ln.
Notice how the TI-89 simplifies this expression.</td><td>
</td></tr>
<tr><td>Now we can take the limit of this expression as n approaches infinity.

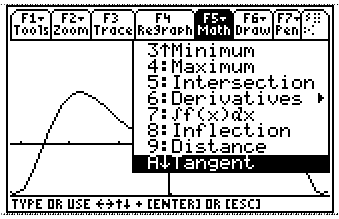
Note that it returns a result of 8/3.</td><td>
</td></tr>
<tr><td>Finally, we can check our work by computing the definite integral $\int_0^2 x^2 dx$:</td><td>
</td></tr>
</table>

5. Tangent lines
This is a topic that the TI-89 handles very easily.

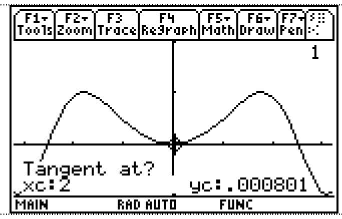
Let's consider the graph of y = sin(x^2/5) over the window [-5,5]x[-1,2], with the graph as shown below:



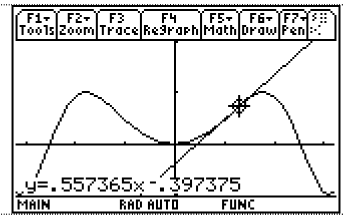
To draw the tangent line and get its equation at any point, we access menu F5:Math and choose option A:Tangent.



When it asks **Tangent at ?**, you can either arrow to the point you want, or just enter it from the keyboard.
Here we enter the number 2 from the keyboard.



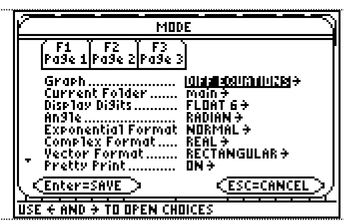
The result of this command draws the tangent line and gives its equation.



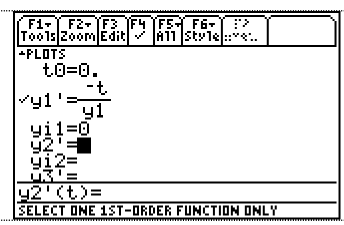
6. Slope Fields

The TI-89 can sketch the slope field for a differential equation.
A slope field is a way to graphically see a differential equation.
There are a few steps to be attended to before we can get the sketch.
NOTE: y1 takes the place of y, and t takes the place of x.

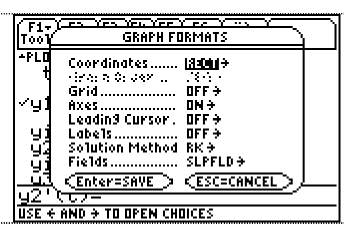
Press the Mode key and make the selections seen to the right.



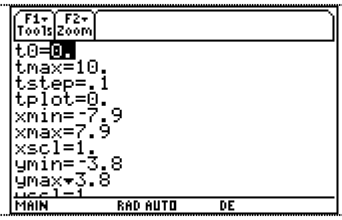
Enter the differential equation in the Y-editor.
Here it is dy/dx = -x/y, which we enter as –t/y1.

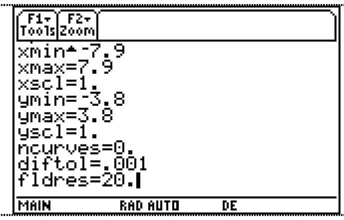


Now choose Format…, which is found under F1, choice 9, and make the selections as shown to the right.

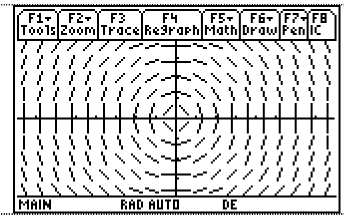


Next, go to Window and select the appropriate choices.

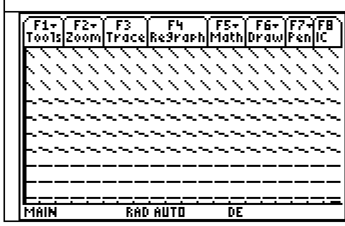
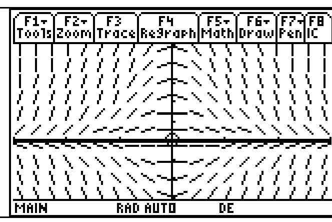
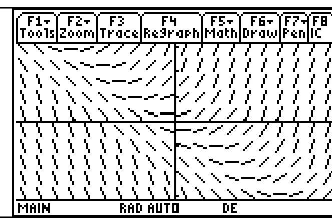




Now you're ready to press the Graph command, and get the desired slope field.

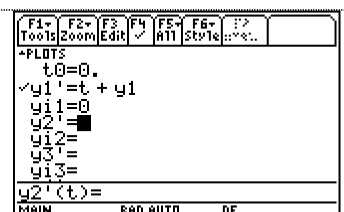


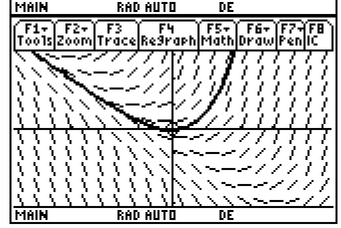
Here are some examples:

dy/dx = -0.1(y-70) Window [0,20]x[60,220]	dy/dx=-2xy Window [-3,3]x[-3,5]	dy/dx = x+y Window [-4.7,4.7] x [-3.1,3.1]
		

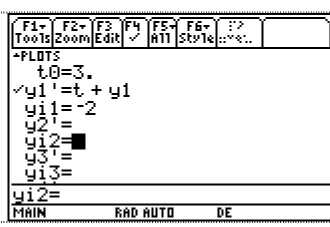
We can see a particular graphical solution, given an initial point.

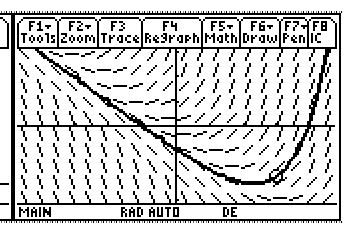
In the previous example, suppose we know the point (0,0) is on the graph of the solution. We make the additional entries in the Y= Editor as shown to the top right, to obtain the graph shown at the bottom right.
The graphical solution to the differential equation is plotted in bold.





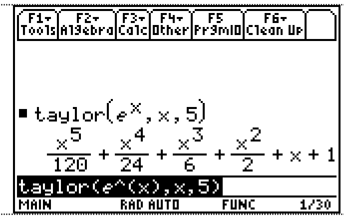
Notice how different the solution looks if we select (3,-2) as an initial point:



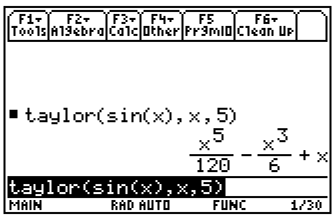


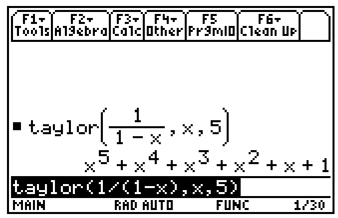
7. Taylor Series

Here is the call we use on a TI-89 to do a Taylor series. This assumes that the series is centered about x=0, which is a Maclaurin series.

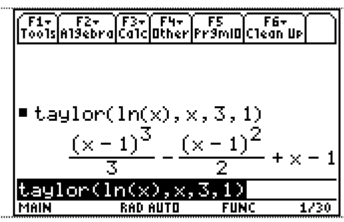


Here are a few other examples:





Here is an example where the series is centered about a number other than x = 0, which in this case we choose as x=1. We choose x=1 because y=ln(x) is not defined for x=0.



Note that this last example shows how we may find a series expansion for $\int e^{x^2} dx$, even though this expression does not have an antiderivative